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On the Dynamics of Inclined Neptune's Trojans

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ABSTRACT

The dynamics of artificial asteroids on the Trojan-like orbits around Neptune is investigated in this paper. We describe the dependence of the orbital stability on the initial semimajor axis a and inclination i by constructing a dynamical map on the $(a, i)-$ plane. Rich details are revealed in the dynamical map, especially a unstable gap at $i = 45°$ is determined and the mechanism triggering chaos in this region is figured out. Our investigation can be used to guide the observations.

Key words: Planets and satellites: Neptune, asteroids, methods: numerical

1 INTRODUCTION

In the restricted three-body model consisting of the Sun, a planet and an asteroid, the equilateral triangular Lagrange equilibrium points (L_4 and L_5) are stable for all planets in our Solar system. Asteroids in the vicinities of L_4 and L_5 are called Trojans after the the group of asteroids found around Jupiter's Lagrange point. Trojan asteroids of Mars and Earth have also been observed while the Trojan-type orbits of Saturn and Uranus have been proved unstable due to the perturbations from other planets. As for Neptune, 6 Trojan-like asteroids were discovered in recent years (IAU: Minor Planet Center, http://www.cfa.harvard.edu/iau/lists/NeptuneTrojans.html). We list their orbital properties in Table 1. The long-term orbital stability of these asteroids has been studied and verified in different papers, e.g. (Marzari et al. 2003; Brasser et al. 2004b; Li et al. 2007). There could be much more Trojan-type asteroids sharing the orbit with Neptune than with Jupiter, both in the sense of number and total mass (Sheppard & Trujillo 2006). Therefore it is worth to investigate the stable region in the whole parameter space. This topic can be found in several papers (Nesvorný & Dones 2002; Dvorak et al. 2007).

Since the L_5 point of Neptune is nowadays in the direction of the Galaxy center and not suitable for asteroid observing, all the asteroids in Table 1 are around the L_4 point. There are reports that the shape and size of the stable regions around L_4 and L_5 points are different from each other (Holman & Wisdom 1993), but further analysis prove that this asymmetry is no more than an artificial effect of asymmetric initial conditions (Nesvorný & Dones 2002; Dvorak et al. 2008). Thus, it is reasonable to study only one of the Lagrange points and expect the other one has the same dynamical behavior.

All the Trojans in Table 1 are on the near-circular orbits (small eccentricities) and two of them have high inclination values. The origin of the high-inclined orbit is an interesting

Table 1. Orbits of Neptune's Trojans. The mean anomaly M is given at epoch TD=20080514. The perihelion argument ω, ascending node Ω and inclination i are in degree (J2000.0).

Designation	M	ω	Ω	i	e	a (AU)
2001 QR322	58.89	158.5	151.6	1.3	0.031	30.262
2004 UP10	339.30	359.3	34.8	1.4	0.027	30.171
2005 TN53	284.82	86.7	9.3	25.0	0.064	30.143
2005 TO74	265.46	304.0	169.4	5.3	0.052	30.151
2006 RJ103	231.55	32.7	120.8	8.2	0.027	30.036
2007 VL305	351.72	215.1	188.6	28.1	0.062	30.007

topic (Li et al. 2007), but in this paper, we will discuss only the stability of inclined orbits and try to find out the possible region (in dynamical sense) where the potential Neptune Trojans could survive for long time.

2 DYNAMICAL MAP

To investigate the effects of inclination on the stability of Trojans, we numerically simulate the evolutions of thousands of test particles on the Trojan-like orbits. The dynamical system consists of the Sun, four jovian planets (Jupiter, Saturn, Uranus and Neptune) and the massless test particles. For each set of initial conditions, a specific inclination value is given and 101 artificial Trojans are initialized around the L_5 point of Neptune as follow: Their eccentricities e_0, ascending nodes Ω_0 and mean anomalies M_0 are exactly the same as the ones of Neptune, but the perihelion arguments ω_0 differs $60°$ from the one of Neptune. Because the Trojans share the same orbit with the planet, they are in the 1:1 mean motion resonance with the planet. The critical argument of this resonance is $\sigma = \lambda - \lambda_N$ where $\lambda = \omega + \Omega + M$ is the mean longitude and the subscript 'N' denotes Neptune.
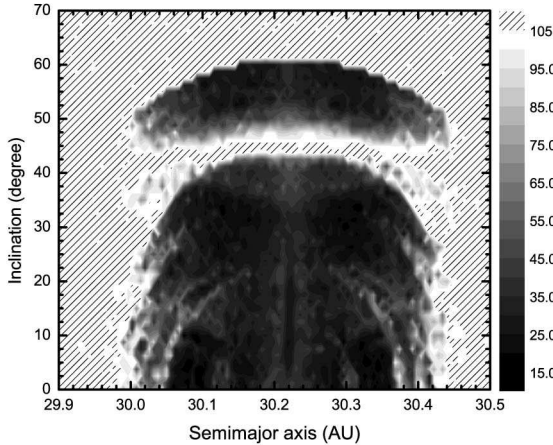
Figure 1. The dynamical map. The spectral number of the resonant argument σ is mapped on the initial (a_0, i_0) plane.

Figure 2. Contour of libration amplitude of the resonant argument. The curves of $\Delta\sigma = 10°, 30°, 50°$ are labeled. The dashed and thick solid curves are for $\Delta\sigma = 20°$ and $70°$, respectively.

In this paper, we always study the case of the trailing Lagrange point (L_5), so that $\sigma_0 = -60°$. The semimajor axes a_0 of test particles are from 29.9 AU to 30.5 AU with an increment 0.006 AU (the osculating semimajor axis of Neptune is 30.14 AU at the starting of simulation). Finally, we vary their inclinations from $0°$ to $70°$ with an increment of $1.25°$. The systems are then integrated to 2.687×10^7 yrs with a Lie-integrator (Hanslmeier & Dvorak 1984). An on-line low-pass digital filter is applied to filter the high-frequency terms in the output and reduce the final data size. We apply spectral analysis to the final data and use the *spectral number* to indicate the regularity of an orbit. In principle, the spectral number (hereafter SN) is the number of peaks in a power spectrum which are higher than a specific threshold. For details of calculating SN, see for example (Ferraz-Mello et al. 2005).

Figure 1 shows the dynamical map on the initial (a_0, i_0) plane. The grey depth indicates the SN of the resonant argument σ. The spectral number is forced to be 100 if it is greater than 100, and those orbits with averaged semimajor axis $\bar{a} \notin [29.9, 30.5]$ (AU) are given a spectral number of 110 (dashed area in Figure 1) since they are surely not inside the 1:1 resonance. Those orbits with small SNs are dominated by few dominating frequencies thus are more regular while the bigger SNs indicate strong noises in the motion and chaotic orbits. To verify the reliability of this regularity indicator, we also integrate hundreds of orbits to the solar system age (4.5G years) using the *hybrid symplectic integrator* in the *Mercury6* integrator package (Chambers 1999). The comparison between the SNs derived from our 2.687×10^7 yrs integrations and the results from the 4.5G years integrations done by the Mercury6 show that orbits with small SNs are generally regular and survive on the Trojan-like orbits in 4.5G years while orbits with SNs higher than ~ 60 will escape from the resonant region and/or be ejected by (collide with) the planets or the Sun.

There are some interesting features in Figure 1 deserving a description.

1) The center of the resonant orbits is at ~ 30.218 AU in terms of the initial semimajor axis while the mean semimajor axis is ~ 30.11 AU. The stable region is distributed
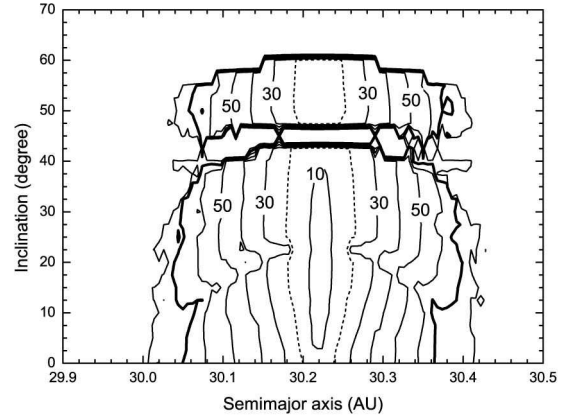
symmetrically with respect to this center. Away from the center, the libration amplitude of the resonant argument $\Delta\sigma$ increases as shown in Figure 2 and the Trojans run on the so-called 'tadpole' orbit. It is reported that no Neptune's Trojan can survive with $\Delta\sigma > 60°$ (Holman & Wisdom 1993; Nesvorný & Dones 2002). Comparing Figure 1 and 2, we can derive the same conclusion. The inner and outer edges of the resonant region seen in Figure 1 $a_0 \sim 30.04$ AU and ~ 30.40 AU are defined by the overlapping of the secondary resonances.

2) The stable orbits with initial inclination as high as $i_0 = 60°$ exist as Figure 1 indicates. This upper limit in fact can be found in a restricted three-body model in which it's $61.7°$ (Brasser et al. 2004a). Different values ($35°$ and $70°$) of such a threshold (Nesvorný & Dones 2002; Dvorak et al. 2007) probably are due to the specific initial conditions of the orbits, the stability criteria and the inadequate sample Trojans in their simulations.

3) The most distinguishable feature in Figure 1 is a gap locating at $i_0 \sim 45°$. It separates the resonant region into two disconnected parts. Therefore we may expect to find two primordial Trojan groups in future, because this gap prohibits Trojans in one region from entering the other through secular diffusion.

4) There are rich details in the resonant region. For example, two less regular regions at low inclination and with $a_0 \sim 30.11$ and ~ 30.31 AU are visible in Figure 1. They arise from the secular resonance ν_{18}, that is, the nodal frequency of the Trojan in these regions are very close to Neptune's nodal frequency, $\dot{\Omega} \sim \dot{\Omega}_N$. Another noteworthy structure is an arc of irregular motion extending from ($a_0 = 30.04$ AU, $i_0 = 0°$) to ($a_0 = 30.15$ AU, $i_0 = 23°$), and the symmetrical arc on the right side also. Although the mechanism behind is not understood very well, the arc is reflected in Figure 2 again as a valley of large libration amplitude.

Up to now all the initial eccentricities of test Trojans are set to be the same as Neptune ($e_0 = 0.006$). In the dynamical evolution, the eccentricities of stable orbits are kept small. In fact for most of stable orbits with $i_0 < 45°$, the maximum eccentricity is smaller than 0.05. Only some
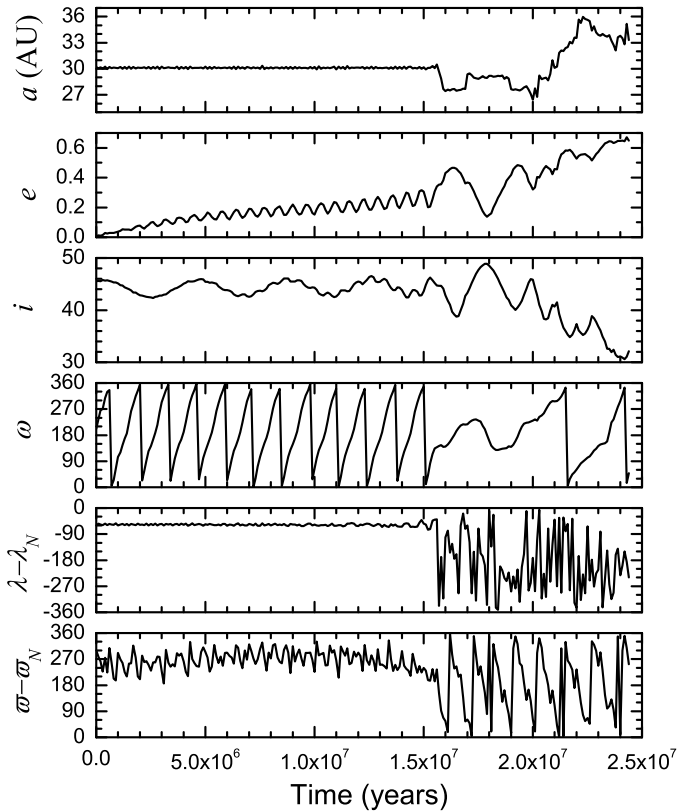
precession of the perihelion may drive the eccentricity up (Murray & Dermott 1999). We check the frequency of the perihelion longitudes of the Trojan and the planets in the system, the result show that the secular resonance ν_8 ($\dot{\varpi} \approx \dot{\varpi}_N$) is responsible for the eccentricity increasing. A proof of this secular resonance is clearly shown in the bottom panel in Figure 3 where $\varpi - \varpi_N$ librates around $\sim 270°$.

This high eccentricity makes the Trojan's perihelion distance $q = a(1 - e) \approx 19.4\,\mathrm{AU}$, which means the Trojan may cross the orbit of Uranus ($a_U = 19.2\,\mathrm{AU}$). But its high inclination makes the probability of close encounter with Uranus very small thus the orbit can be still safe. However, the high eccentricity also makes another secular resonance possible, the Kozai resonance (Kozai 1962), in which the perihelion argument ω librates while the eccentricity and inclination undergo variations such that the quantity $H_K = \sqrt{1 - e^2}\cos i$ remains constant. These can be found after $T = 1.57 \times 10^7$ yrs in the 2nd, 3rd and 4th panel of Figure 3.

In Kozai resonance, when the eccentricity increases the inclination decreases. Consequently the probability of close encounter with Uranus or other planets is enhanced significantly, and such close encounters make the Trojan's orbit unstable, as Figure 3 shows.

4   CONCLUSION

The dependence of the stability of Neptunian Trojans on their inclinations is investigated and shown by a dynamical map. A gap of unstable orbits with initial inclination of $\sim 45°$ has been discovered. The mechanism responsible for this unstable gap, a combined effects from the ν_8 secular resonance and the Kozai resonance has been figured out.

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Figure 3. A typical orbit in the unstable gap in Figure 1. The initial orbital elements of this orbit are $a_0 = 30.218\,\mathrm{AU}$, $e_0 = 0.006, i_0 = 46.25°$. From top to bottom, the panel shows the evolution of the semimajor axis, eccentricity, inclination, perihelion argument, resonant argument and the difference between the perihelion longitudes of the Trojan and of Neptune. The integration of this orbit was terminated at 2.45×10^7 yrs when this Trojan was ejected by a close encounter with Uranus.

Trojans with $i_0 > 45°$ may be on eccentric orbits, but the eccentricities are still limited by $e \sim 0.1$.

3   MOTION IN THE GAP

The most prominent trait in the dynamical map (Figure 1) is the gap at $i_0 \sim 45°$. We applied the frequency analysis to orbits starting from the gap and have figured out the mechanism causing the chaotic motion. In Figure 3, we illustrate a typical orbit in the gap and use this as an example to explain how an orbit in the gap evolves.

As shown in Figure 3, the resonant argument $\sigma = \lambda - \lambda_N$ librates with a small amplitude around the Lagrange point L_5 with $\sigma \in (-65°, -52°)$ before 1.57×10^7 yrs. This libration implies that the Trojan is in the 1:1 mean motion resonance with Neptune. Thanks to the protection of the resonance, the evolutions of other orbital elements during this period are also regular. For example the semimajor axis is nearly constant and the inclination variation around $\sim 44°$ with an amplitude of only $\sim 4°$. But there is one exception, the eccentricity of the Trojan keeps increasing during this period and, the eccentricity reaches $e = 0.355$ at $T = 1.57 \times 10^7$ yrs. We know that the secular resonance related to the